

Yet More on the “Universal” Quantum Area Spectrum *

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May 18, 2010

Abstract

We briefly comment on the quantum area spectra of black holes, paying particular attention to the size of the spacing between adjacent spectral levels. It has previously been conjectured that this spacing is uniform with a universal value of 8π in Planck units. In spite of a recent claim to the contrary [1], we argue that this particular value remains, by far, the most qualified candidate for a universal area gap.

* Alternative Title: *Comment on “A Note on the Lower Bound of Black Hole Area Change in Tunneling Formalism”*

Ever since Bekenstein [2] proposed that a black hole should have a quantum spectrum for its horizon area, there has been a significant amount of debate on the topic. The discourse has covered both the form of the spectra — evenly spaced or otherwise (*e.g.*, [3]) — and, if uniformly spaced as per Bekenstein’s original account, then the size of the gap between adjacent levels (*e.g.*, [4]). Let us, for the sake of current considerations, take the evenly spaced form as a given and focus our attention on the second issue.

Formally speaking, we are then asking as to the size of the dimensionless parameter γ when the spectrum for the area operator ($A_n \equiv \langle \hat{A} \rangle$) is expressed as

$$A_n = A_0 + \gamma l_P^2 n \quad \text{where } n = 0, 1, 2, \dots \quad (1)$$

Here, l_P is the Planck length¹ and A_0 allows for the possibility of a non-vanishing zero-point term. Given that l_P is the length scale at which quantum gravity sets in, it is naturally expected that γ is of the order of unity. However, without further inputs, it is difficult to be more concise on its value.

If one wishes to pursue the matter, the two viable avenues (that the current author is aware of) are to either appeal to a specific theory of quantum gravity or to follow some semi-classical line of reasoning. An interesting example of the second option is seen in a very recently submitted letter by Banerjee *et. al.* [1]. Closely following Bekenstein’s legendary treatment of a black hole absorbing a quantum particle in the vicinity of its horizon [5], these authors arrive at the following bound (although translated into our conventions):

$$\gamma \geq \frac{8\pi}{\hbar} \epsilon \Delta \hat{X} \langle \hat{E} \rangle \quad (2)$$

Here, \hat{E} and \hat{X} are quantum operators for the particle’s energy and displacement from the horizon, whereas ϵ is a dimensionless “fudge factor” reflecting the rather ambiguous nature of assimilation by a black hole. (Meanwhile, Δ and $\langle \rangle$ always maintain their usual meanings of quantum uncertainty and expectation value.)

The cited authors then appeal to a variation [6, 7] of the often-discussed tunneling mechanism [8] (although knowledge of Hawking’s thermal spectrum for black hole radiation [9] and a text book on statistical mechanics would have served just as well) to deduce that

$$\langle \hat{E} \rangle = T_H \quad (3)$$

with T_H denoting the Hawking temperature of the black hole. Additionally, for later use, $\langle \hat{E}^2 \rangle = 2T_H^2$, so that

$$\Delta \hat{E} = \sqrt{\langle \hat{E}^2 \rangle - \langle \hat{E} \rangle^2} = T_H \quad (4)$$

Substituting Eq. (3) into (2), they then obtain

$$\gamma \geq \frac{8\pi}{\hbar} \epsilon \Delta \hat{X} T_H \quad (5)$$

¹We have committed to four dimensions, with the usual disclaimer that generalizations are readily attainable. Also note that, for future reference, the speed of light and the Boltzmann constant are always set to unity.

So far, so good.

The critical steps are now upon us. The stated authors use the quantum uncertainty principle $\Delta\hat{X}\Delta\hat{P} \geq \hbar$ (with P representing the particle's momentum operator) and the obvious bound $\Delta\hat{E} \geq \Delta\hat{P}$ (or taken as an equality for a massless particle) to argue that

$$\Delta\hat{X} \geq \frac{\hbar}{\Delta\hat{E}} \quad (6)$$

and then, by virtue of Eq. (4),

$$\Delta\hat{X} \geq \frac{\hbar}{T_H} . \quad (7)$$

Substituting this result into Eq. (5), they end up with

$$\gamma \geq 8\pi\epsilon . \quad (8)$$

Had the authors stopped here, it would be a perfectly reasonable deduction. And, given the ambiguity that is inherent through ϵ , almost certainly a true observation (not to mention reminiscent of Bekenstein's [5]). However, they then proceed to *double down*² on their analysis by attempting to fix ϵ via the first law of black hole mechanics [10]. More to the point, this law can be expressed (in its simplest form) as

$$T_H \frac{\delta A}{4l_P^2} = \delta M , \quad (9)$$

where M is the mass or rest energy of the black hole. Let us rewrite this by attributing δM to the absorption of the previously discussed particle and then suitably quantizing:

$$T_H \frac{\gamma}{4} = \langle \hat{E} \rangle . \quad (10)$$

Substituting Eq. (3) into the above expression — which is reasonable facsimile of that used by the discussed authors³ — we finally arrive at the authors claim of

$$\gamma = 4 . \quad (11)$$

Or, in other words, $\epsilon = 1/2\pi$ with Eq. (8) now taken to be a strict equality.

Although this all seems reasonable enough, the logic is unfortunately flawed, as we now explain: The identification $\delta M = \langle \hat{E} \rangle = T_H$ might be true if one were endeavoring to calculate the thermal or *canonical* fluctuations in the horizon area. Indeed, the very notion of evaluating variations in energy at a fixed value of temperature (in this case $T = T_H$) only makes sense in a canonical setting. However, this is not what is meant (at least not purposefully) when one talks about the quantum area spectrum of a black hole. Eq. (1) is, rather, meant to be the *fundamental* quantum spectrum for a black hole, which implies that the appropriate setting is, in actuality, a *microcanonical* one.⁴ That is, one

²For any confused readers, this terminology is borrowed from the casino game “Black Jack”. Doubling down is a strategic option that enables one to double his or her potential winnings at the cost of doubling the potentiality for loss.

³Actually, the authors of [1] take the change in black hole mass to be $\Delta\hat{E}$, which does not seem quite right. However, since $\Delta\hat{E} = \langle \hat{E} \rangle = T_H$, their choice amounts to the same thing.

⁴Although long advocated by the current author (*e.g.*, [11]), G. Gour was the first to emphasize the importance of distinguishing between the canonical and microcanonical contributions to the area spectrum [12].

should fix the energy of the system and then inquire as to how the spectral levels of a given quantity (in this case the area) are distributed. In all practicality, thermal fluctuations would be present and, likely, blur the original spectral lines. But such fluctuations are not relevant to questions about the fundamental nature of the quantized geometry.

Then, is it possible to be more definitive about the spacing parameter γ ? To this end, let us start with the simplest case of a Schwarzschild black hole, for which there is only one relevant length scale: the horizon radius or, equivalently, the inverse of the Hawking temperature. Hence, it can be expected on dimensional grounds that $\delta M \sim T_H$, so that γ is of order unity. Now, insofar as γ truly has the status of a universal parameter (in accordance with Occam’s razor if nothing else), it would follow that the very same order-unity value carries through to more elaborate scenarios; including black holes of the spinning, charged and/or “hairy” variety.⁵

We can not, however, be more specific than this without further inputs, which generally (if not inevitably) necessitates further assumptions about what constitutes a viable quantum theory of gravity. Nevertheless, many treatments have independently produced the same value of $\gamma = 8\pi$ — see [16] and references therein. More recent examples include a quantization procedure proposed by Ropotenko [17], a refinement thereof [15] and, perhaps most persuasively, Maggiore’s reinterpretation [18]⁶ of the renowned Hod conjecture [4]. In addition, it is probably worth mentioning the “emergent gravity” conjectures of Padmanabhan [22] and Verlinde [23]. In this context, one finds that the unique choice of $\Delta S = 2\pi$ (where ΔS is the minimal change in the entropy that is responsible for gravity) leads almost miraculously⁷ to Newton’s second law of mechanics and Newton’s law of gravitation, amongst others. Translated in terms of the black hole area–entropy law [9, 24], this becomes $\Delta A = 8\pi l_P^2$ or, once again, $\gamma = 8\pi$.

It may be true that all of the studies returning $\gamma = 8\pi$ are limited contextually by their scope and/or technically through their assumptions (both explicit and implied). So it is still feasible that all of these studies are simply wrong and γ is not 8π after all. For that matter, γ may not even be universal. Nevertheless, taken as a whole, the overall body of evidence is pretty compelling.

A possible counter-example, as pointed out by Banerjee *et. al.* [1], could be Hod’s generalization [25] of Bekenstein’s calculation [5]. After revising the analysis to that of a charged particle being absorbed by a charged (Reissner–Nordstrom) black hole, Hod advocated for the contrary result of $\gamma = 4$. However, what Hod actually formulated was a lower bound, so the more accurate statement is $\gamma \geq 4$, which is in no way contradictory.

In conclusion, we assert that (I) $\gamma = 8\pi$ is still, by far, the most qualified candidate for a universal area spacing (if any) and (II) Banerjee *et. al.* have provided no evidence in [1] that casts dispersion upon the first claim.

⁵Even more elaborate is when the gravitational theory differs from Einstein’s. In this case, γ should be regarded as the spacing between the spectral levels of the operator $\hat{S}_W/4l_P^2$, where S_W is meant to represent Wald’s geometric or Noether-charge entropy [13]. Given a generic theory of gravity, the spectrum for this quantity is, as made clear in [14, 15], the unequivocal analogue to the area spectrum.

⁶Also see, for instance, [19–21].

⁷Or, perhaps, coincidentally.

Acknowledgments

The author's research is financially supported by the University of Seoul.

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